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GENERAL ARRANGEMENT OF REGIMES FOR SPATIAL LOCAL FLOWS

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Different local features at the surface of a body are breaks or sharp changes in boundary conditions, separation or joining of a flow, irregularities, etc., and they may have a marked effect on local and global characteristics of flow over it [1]. This situation stimulates continued interest towards to flow in local regions, which apart from considerable practical importance, often exhibit considerable theoretical novelty (see, e.g., [2-6], where a systematic study was carried out of planar local regions of flow). However, the majority of local regions are spatial, and whereas in studying flat regions considerable success have been achieved, for spatial regions only individual solutions have been obtained, often using considerable simplifications [7-19]. In addition, due to the absence of systematic studies it is difficult to determine the boundaries for existence of different flow regimes in local spatial regions, and limiting transitions which make it possible to changeover from one flow regime to another. In this work systematic studies are carried out for flow regimes in local spatial regions for each of the boundary problems formulated, the main properties of their solution are studied, and a general classification for the arrangement of flow regimes is built up.

1. Consideration is given to flow over a flat semi-infinite plate by a uniform subsonic or supersonic flow of viscous gas with Mach numbers $(M_\infty^2 - 1) \sim O(1)$ or more, but for precritical Reynold's numbers. Let there be in the surface of the plate at a certain distance ℓ from its leading edge a small spatial protuberance or hollow (Fig. 1). A steady-state solution of the Navier–Stokes equations is constructed for a spatial region of disturbed laminar flow with $\mathrm{Re}_\infty = \rho_\infty u_\infty l/\mu_\infty = \varepsilon^{-2}$ tending towards infinity. Here ρ_∞, u_∞, and μ_∞ are values of density, velocity, and dynamic viscosity coefficient in an undisturbed uniform running flow. Subsequently we shall use only dimensionless values, and for this all linear dimensions are related to ℓ, pressure p is related to $\rho_\infty u_\infty^2$, enthalpy h is related to u_∞^2, and the rest of the flow functions are related to their values in an undisturbed uniform running flow.

Considering the dimensions of a small irregularity, it is assumed that its typical thickness a in order of magnitude is less or equal to the typical width of an undisturbed boundary layer on a plate in this area, i.e., $a \lesssim \delta \sim O(\varepsilon)$, and its typical extent b in order of value is greater or equal to a and less than or equal to unity, i.e., $a \leqslant b \leqslant 1$. The nature of the irregularity width c in order of magnitude may be greater or equal to a, i.e., $c \geqslant a$. With a > b or a > c flow may have the same features as that with a ~ b or a ~ c, and only the longitudinal or transverse dimensions of the disturbed flow region will be determined by the value of a. It is evident that a, b, c > ε^2 (for flow regions in which one of the characteristic dimensions is commensurate in order of value to the typical length of free flow of a gas molecule $\sim O(\varepsilon^2)$, Navier–Stokes equations will not be valid), i.e., characteristic thickness a, extent b, and width c of an irregularity are satisfied by the relationships

$$\varepsilon^2 < a \leqslant \varepsilon,\ a \leqslant b \leqslant 1,\ a \leqslant c. \qquad (1.1)$$

This means that the test region for measurement of values of a, b, and c is limited by boundaries of truncated pyramid ABCDEFGH (Fig. 2). Among irregularities with characteristic dimensions (1.1) consideration is only given to those which initiate considerable local pressure gradients $\partial p/\partial x > 1$ or $\partial p/\partial Z > 1$ or for which in the disturbed flow regions convective

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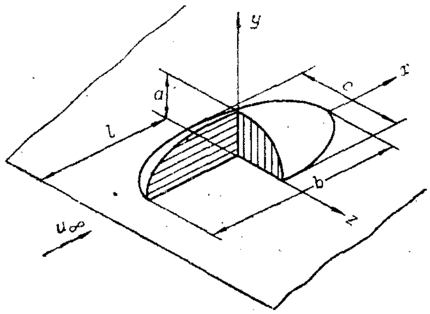

Fig. 1

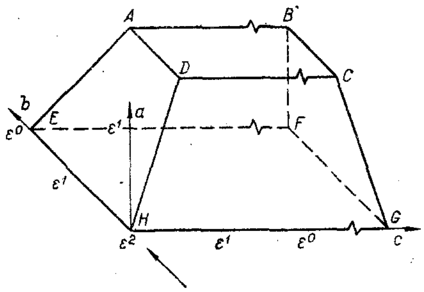

Fig. 2

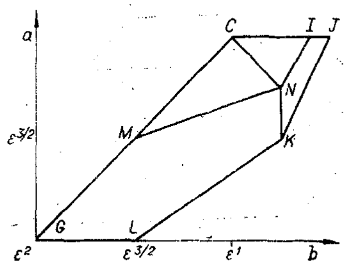

Fig. 3

and diffusion terms of Navier–Stokes equations (e.g., $\rho u \partial u/\partial x > 1$ or $\varepsilon^2 \mu \partial^2 u/\partial y^2 > 1$) are large.

2. First let c > b, i.e., consideration is given to flow over irregularities of the trench or embankment type $\varepsilon^2 < a \leqslant b < c$ which are wide in the transverse direction. It is evident that in this case spreading to the sides will be small and the basic estimates for flow functions in disturbed regions over such irregularities will conform with estimates for the case of flow over flat irregularities.

If the thickness of an irregularity a is so small that disturbance of the flow functions is only created due to reaction of the irregularity with the boundary subsonic shear part of the undisturbed boundary layer on the plate, then in a layer of nonlinear disturbances close to the surface of the irregularity, where disturbance of the longitudinal velocity in order of value is equal to the velocity itself $u \sim \Delta u \sim \Delta p^{1/2}$,

$$u \sim \Delta u \sim O(a/\varepsilon),\ \Delta p \sim O(a^2/\varepsilon^2). \tag{2.1}$$

In the limiting case for "thick" irregularities, when pressure disturbance is created due to action of the irregularity with an undisturbed uniform running flow, from normal theory for small disturbances it follows that

$$\Delta p \sim O(a/b). \tag{2.2}$$

In the transitional case, when both estimates are valid for disturbance of pressure (2.1) and (2.2), and disturbance of the flow function is created due to reaction of the irregularity with all of the boundary layer on the plate,

$$ab \sim O(\varepsilon^2). \tag{2.3}$$

If the layer of nonlinear disturbances is viscous, then its typical thickness $\delta_1 \sim O(\varepsilon b^{1/2}/\Delta p^{1/4})$, and taking account of the fact that close to the surface of the body flow functions change in proportion to the distance from it (e.g., $u \sim y/\varepsilon$), for all regimes of flow over irregularities the following will be fulfilled

$$\delta_1 \sim O(\varepsilon b^{1/3}),\ \Delta p \sim O(b^{2/3}). \tag{2.4}$$

Estimates (2.1)-(2.4) make it possible to build up disturbed flow in the plane x, y over irregularities that are planar or extensive in the transverse direction. These studies were carried out in [3] where a classification scheme is given for the corresponding flow regimes (Fig. 3).

The region for change in thickness a and extent b for irregularities that are wide in the transverse direction is bounded by polygon GCJKL. Lines $LK(a \sim O(\varepsilon b^{2/3}))$ and $KJ(a \sim O(b^2))$ cut off flow regimes with which small pressure gradients $\partial p/\partial x$ are initiated.

In region GCNKL disturbance of low functions is only created due to reaction of an irregularity with the boundary-wall part of the boundary layer on a plate, and here estimate (2.1) is valid.

In region CJKN disturbance of the flow functions is created due to reaction of the irregularity with a uniform running flow, and here estimate (2.2) operates.

On lines CN $(a \sim O(\varepsilon^2/b))$ and NK $(b \sim O(\varepsilon^{3/4}))$ a change in displacement thickness of the boundary-wall region of disturbed flow occurs as a result of reaction of the irregularity with the boundary-wall part of the boundary layer on a plate, and pressure disturbance is created due to reaction of the effective thickness of the irregularity (strictly the thickness of the irregularity plus the displacement thickness of the boundary-wall region) with a uniform running flow.

On lines MN $(a \sim O(\varepsilon b^{1/3}))$ and $NI (a \sim O(b^{5/3}))$ the irregularity causes viscous nonlinear disturbances, and here disturbance of frictional stress τ_{xy} in order of value equals its basic value in an undisturbed boundary layer at the plate surface.

On line GM $(a \sim O(b))$ flow in the undisturbed region is described by a Stokes equation, and at point M $(a \sim b \sim O(\varepsilon^{3/2}))$ it is described by Navier–Stokes equations for an incompressible gas [20]. On line MN a compensation regime is realized for flow over irregularities [21], and at point $N (a \sim O(\varepsilon^{5/4}),\ b \sim O(\varepsilon^{3/4}))$a free reaction regime is realized [2]. On line NI flow over "thick" irregularities is described by Prandtl boundary layer equations for an incompressible gas with prescribed pressure distribution [22].

Above line MNI and in region NCIN irregularities cause nonviscous nonlinear disturbances, and in this way close to the surface of irregularities it is necessary in addition to consider a viscous sublayer. Lower down, in region GMNIJKL, irregularities cause viscous linear disturbances.

An estimate for velocity component w in the transverse direction z is obtained from an equation for conservation of a transverse pulse $\rho u \partial w/\partial x \sim \partial p/\partial z$, and taking account of estimates (2.4) in a layer of viscous nonlinear disturbances:

$$w \sim O(b^{4/3}/c),\ \partial w/\partial z \sim O(b^{4/3}/c^2) \sim (b^2/c^2)\partial u/\partial x. \tag{2.5}$$

Relationships (2.5) indicate that in the case being considered with c > b for all of the regimes being studied for flow over irregularities the whole set of equations describing the spatial region of disturbed flow breaks down into a set of equations describing flow over flat sections of irregularities containing coordinate z as a parameter, and into an equation for conservation of a transverse pulse (linearized in relation to velocity component w, i.e., without term $\rho w \partial w/\partial z$ in the convective operator), which may be solved separately.

It is evident that flow over irregularities that are wide in the transverse direction exhibits the same features as for flow over flat irregularities.

3. With an increase in the width of an irregularity c up to c ~ O(b) estimates (2.1)-(2.5) remain in force, only now in the layer viscous nonlinear disturbances of transverse velocity component w are equal in order of value to the longitudinal velocity component u:

$$w \sim u \sim O(b^{1/3}), \tag{3.1}$$

stress component τ_{yz} is equal in order of value to stress τ_{xy}, and the set of equations describing the spatial region of disturbed flow is not broken down.

Estimates (2.1)-(2.4) and (3.1) make it possible to construct a solution for Navier–Stokes equations for the case of flow over narrow irregularities of the circular pit and mound type $\varepsilon^2 < a \leqslant b \sim c \leqslant 1$. In layer 3 viscous nonlinear disturbances introduce the following independent variables and asymptotic expansions for flow functions:

$$\begin{gathered} x = bx_3,\ y = \varepsilon b^{1/3} y_3,\ z = bz_3, \\ u = b^{1/3}u_3 + \ldots,\ v = (\varepsilon/b^{1/3})v_3 + \ldots,\ w = b^{1/3}w_3 + \ldots, \\ p = 1/\gamma M_\infty^2 + b^{2/3}p_3 + \ldots,\ \rho = \rho_w + \ldots,\ \mu = \mu_w + \ldots \end{gathered} \tag{3.2}$$

Here γ is the ratio of specific heat capacities; index w relates to values at the plate surface in the point where the irregularity is located.

Substitution of expansion (3.2) in Navier–Stokes equations and completion of the limiting transition with $\varepsilon \to 0$ indicates that to a first approximation region 3 is described by complete Navier–Stokes equations for an incompressible gas with $a \sim b \sim c \sim O(\varepsilon^{3/2})$

$$\begin{gathered} \frac{\partial u_3}{\partial x_3} + \frac{\partial v_3}{\partial y_3} + \frac{\partial w_3}{\partial z_3} = 0,\ \rho_w\left(u_3\frac{\partial u_3}{\partial x_3} + v_3\frac{\partial u_3}{\partial y_3} + w_3\frac{\partial u_3}{\partial z_3}\right) + \frac{\partial p_3}{\partial x_3} = \mu_w \nabla^2 u_3, \\ \rho_w\left(u_3\frac{\partial v_3}{\partial x_3} + v_3\frac{\partial v_3}{\partial y_3} + w_3\frac{\partial v_3}{\partial z_3}\right) + \frac{\partial p_3}{\partial y_3} = \mu_w \nabla^2 v_3, \\ \rho_w\left(u_3\frac{\partial w_3}{\partial x_3} + v_3\frac{\partial w_3}{\partial y_3} + w_3\frac{\partial w_3}{\partial z_3}\right) + \frac{\partial p_3}{\partial z_3} = \mu_w \nabla^2 w_3 \end{gathered} \tag{3.3}$$

or Stokes equations with $\varepsilon^2 < a \sim b \sim c < \varepsilon^{3/2}$

$$\frac{\partial u_3}{\partial x_3} + \frac{\partial v_3}{\partial y_3} + \frac{\partial w_3}{\partial z_3} = 0, \ \frac{\partial p_3}{\partial x_3} = \mu_w \nabla^2 u_3, \ \frac{\partial p_3}{\partial y_3} = \mu_w \nabla^2 v_3,$$
$$\frac{\partial p_3}{\partial z_3} = \mu_w \nabla^2 w_3, \ \nabla^2 = \frac{\partial^2}{\partial x_3^2} + \frac{\partial^2}{\partial y_3^2} + \frac{\partial^2}{\partial z_3^2}. \qquad (3.4)$$

At the surface of the irregularity $y_3 = f(x_3, z_3)$ normal conditions should be fulfilled for attachment and nonflow

$$u_3 = v_3 = w_3 = 0 \ (y_3 = f(x_3, z_3)), \qquad (3.5)$$

external boundary conditions are obtained from combination with a solution for an undisturbed boundary layer on a plate

$$u_3 \to Ay_3, \ v_3, \ w_3, \ p_3 \to 0 \ (x_3^2 + y_3^2 + z_3^2 \to \infty), \qquad (3.6)$$

where $A = (\partial u_0/\partial y_2)_w$ [$y = \varepsilon y_2$, $u_0(y_2)$ is the profile of velocity u in an undisturbed boundary layer on a plate]. It is well known that (3.3), (3.5), (3.6) or (3.4)-(3.6) are boundary problems of the elliptical type.

If $\varepsilon^{3/2} < b \leqslant 1$, then flow in layer 3 to a first approximation is described by Prandtl spatial boundary layer equations for an incompressible gas

$$\frac{\partial u_3}{\partial x_3} + \frac{\partial v_3}{\partial y_3} + \frac{\partial w_3}{\partial z_3} = 0, \ \rho_w \left(u_3 \frac{\partial u_3}{\partial x_3} + v_3 \frac{\partial u_3}{\partial y_3} + w_3 \frac{\partial u_3}{\partial z_3}\right) + \frac{\partial p_3}{\partial x_3} = \mu_w \frac{\partial^2 u_3}{\partial y_3^2},$$
$$\frac{\partial p_3}{\partial y_3} = 0, \ \rho_w \left(u_3 \frac{\partial w_3}{\partial x_3} + v_3 \frac{\partial w_3}{\partial y_3} + w_3 \frac{\partial w_3}{\partial z_3}\right) + \frac{\partial p_3}{\partial z_3} = \mu_w \frac{\partial^2 w_3}{\partial y_3^2}. \qquad (3.7)$$

At the surface of irregularities whose thickness in order of value equals the thickness of layer 3 ($a \sim O(\varepsilon b^{1/3})$), boundary conditions (3.5) are fulfilled, and for "thick" irregularities with $\varepsilon b^{1/3} < a \sim O(b^{5/3})$ the following boundary conditions are fulfilled

$$u_3 = v_3 = w_3 = 0 \ (y_3 = 0). \qquad (3.8)$$

Here coordinate y_3 is reckoned along the normal from the surface of the irregularity.

Initial boundary conditions are obtained from combination with a solution for an undisturbed boundary layer on a plate

$$u_3 \to Ay_3, \ v_3, \ w_3, \ p_3 \to 0 \ (x_3 \to -\infty, \ z_3 \to \pm\infty), \qquad (3.9)$$

and in order to find external boundary conditions it is necessary to consider in addition region 2 whose characteristic thickness $y \sim O(b)$ with $\varepsilon^{3/2} < b < \varepsilon$ or $y \sim O(\varepsilon)$ with $\varepsilon \leqslant b \leqslant 1$. Therefore, in region 2 in the first case the following independent variables and asymptotic expansions of flow functions are valid:

$$x_2 = x_3 = x/b, \ y_2 = y/b, \ z_2 = z_3 = z/b,$$
$$u = (b/\varepsilon)Ay_2 + (\varepsilon/b^{1/3})u_{22} + \ldots, \quad v = (\varepsilon/b^{1/3})v_{22} + \ldots, \ \rho = \rho_w + \ldots,$$
$$w = (\varepsilon/b^{1/3})\, w_{22} + \ldots, \quad p = 1/\gamma M_\infty^2 + b^{2/3} p_2 + \ldots, \qquad (3.10)$$

and in the second case variables and expansions in the form

$$x_2 = x_3 = x/b, \ y_2 = y/\varepsilon, \ z_2 = z_3 = z/b,$$
$$u = u_0(y_2) + b^{1/3}u_{21} + b^{2/3}u_{22} + \ldots, \ v = b^{2/3}v_{21} + (\varepsilon/b^{1/3})v_{22} + \ldots,$$
$$w = b^{2/3}w_{22} + \ldots, \ p = 1/\gamma M_\infty^2 + b^{2/3}p_2 + \ldots,$$
$$\rho = \rho_0(y_2) + b^{1/3}\rho_{21} + b^{2/3}\rho_{22} + \ldots, \ h = h_0(y_2) + b^{1/3}h_{21} + \ldots, \qquad (3.11)$$

are valid, where profiles of flow functions in an undisturbed boundary on a plate are labeled with index 0. Substitution of expansions (3.10) of (3.11) in Navier–Stokes equations and completion of the limiting transition with $\varepsilon \to 0$ and $\varepsilon^{3/2} < b \leqslant 1$ indicates that in both cases flow in region 2 to a first approximation will be described by linearized Euler equations relating to the running flow [$u = (b/\varepsilon)Ay_2$ or $u = u_2(y_0)$]. In addition, with $\varepsilon^{3/2} < b < \varepsilon^{3/4}$ use of expansions (3.10) or (3.11) leads to the same basic result

$$A\rho_w v_{22} + \partial p_2/\partial x_2 \to 0 \ (y_2 \to 0). \qquad (3.12)$$

The combination of expansions in regions 2 and 3 with the use of relationship (3.12) makes it possible to obtain external boundary conditions

$$u_3 \to Ay_3,\ w_3 \to 0,\ A\rho_w v_3 + \partial p_3/\partial x_3 \to 0\ (y_3 \to \infty). \tag{3.13}$$

The boundary problem (3.5), (3.7), (3.9), and (3.13) describes a spatial compensation regime of flow over irregularities with $a \sim O(\varepsilon b^{1/3})$, $\varepsilon^{3/2} < b < \varepsilon^{3/4}$, $c \sim O(b)$, whose important difference from the corresponding planar regime for flow [21] is propagation of disturbances upwards through the flow [19].

With $b \sim O(\varepsilon^{3/4})$ a linearized set of Euler equations, describing flow in region 2, allows partial integration

$$p_2 = p_2(x_2,\ z_2),\ u_{21} = D du_0/dy_2,\ v_{21} = -u_0 \partial D/\partial x_2,\ D = D(x_2,\ z_2). \tag{3.14}$$

Combination of expansions in regions 2 and 3 with the use of relationship (3.14) gives external boundary conditions in the form

$$u_3 \to A(y_3 + D),\ w_3 \to 0\ (y_3 \to \infty). \tag{3.15}$$

For "thick" irregularities with $a \sim O(b^{5/3})$, $\varepsilon^{3/4} < b \leqslant \varepsilon^{3/5}$ in region 2 the following independent variables and asymptotic expansions of flow functions are introduced:

$$\begin{gathered} x_2 = x_3 = x/b,\ y = \varepsilon y_2 + b^{5/3} f(x_2,\ z_2) + \ldots,\ z_2 = z_3 = z/b, \\ u = u_0(y_2) + b^{2/3} u_2 + \ldots,\ v = b^{2/3} v_2 + \ldots,\ w = b^{2/3} w_2 + \ldots, \\ p = 1/\gamma M_\infty^2 + b^{2/3} p_2 + \ldots,\ \rho = \rho_0(y_2) + b^{2/3}\rho_2 + \ldots, \\ h = h_0(y_2) + b^{2/3} h_2 + \ldots \end{gathered} \tag{3.16}$$

Substituting of expansions (3.16) in Navier–Stokes equations and completion of the limiting transition with $\varepsilon \to 0$ and $\varepsilon^{3/4} < b \leqslant \varepsilon^{3/5}$ indicates that to a first approximation flow in region 2 will again be described by linearized Euler equations, from which it emerges that

$$p_2 = p_2(x_2,\ z_2),\ v_2 = u_0 \partial f/\partial x_2, \tag{3.17}$$

and combination of expansions in regions 2 and 3 leads to external boundary conditions

$$u_3 \to Ay_3,\ w_3 \to 0\ (y_3 \to \infty). \tag{3.18}$$

Now in order to determine pressure disturbances with $\varepsilon^{3/4} \leqslant b \leqslant \varepsilon^{3/5}$ it is necessary to consider disturbed region 1 of an equilibrium running flow, where new independent variables and asymptotic expansions of flow functions are valid

$$\begin{gathered} x_1 = x_2 = x_3 = x/b,\ y_1 = y/b,\ z_1 = z_2 = z_3 = z/b, \\ u = 1 + b^{2/3} u_1 + \ldots,\ v = b^{2/3} v_1 + \ldots,\ w = b^{2/3} w_1 + \ldots,\ \rho = 1 + b^{2/3}\rho_1 + \ldots, \\ p = 1/\gamma M_\infty^2 + b^{2/3} p_1 + \ldots,\ h = 1/(\gamma - 1) M_\infty^2 + b^{2/3} h_1 + \ldots \end{gathered} \tag{3.19}$$

Substitution of expansions (3.19) in Navier–Stokes equations and completion of the limiting transition with $\varepsilon \to 0$ and $\varepsilon^{3/4} \leqslant b \leqslant \varepsilon^{3/5}$ indicates that to a first approximation pressure disturbance in region 1 is described by solution of boundary problem

$$\begin{gathered} \frac{\partial^2 p_1}{\partial y_1^2} + \frac{\partial^2 p_1}{\partial z_1^2} = (M_\infty^2 - 1)\frac{\partial^2 p_1}{\partial x_1^2},\ p_1 \to 0\ (x_1^2 + y_1^2 + z_1^2 \to \infty), \\ p_1(x_1,\ 0,\ z_1) = p_2(x_2,\ z_2) = p_3(x_3,\ z_3), \end{gathered} \tag{3.20}$$

for which the internal boundary condition with $b \sim O(\varepsilon^{3/4})$ is

$$\partial p_1/\partial y_1 = \partial^2 D/\partial x_1^2\quad (y_1 = 0) \tag{3.21}$$

or

$$\partial p_1/\partial y_1 = -\partial^2 f/\partial x_1^2\quad (y_1 = 0) \tag{3.22}$$

with $\varepsilon^{3/4} < b \leqslant \varepsilon^{3/5}$.

Consequently, with $a \sim O(\varepsilon^{5/4})$, $b \sim c \sim O(\varepsilon^{3/4})$ flow over irregularities is described by combined solution of boundary problems (3.5), (3.7), (3.9), (3.15) and (3.20), (3.21). This region for flow over irregularities is a spatial analog of a two-dimensional regime for free reaction [2], and its individual properties have been studied in [9, 10, 13-15].

For the case of flow over "thick" irregularities with $a \sim O(b^{5/3})$, $\varepsilon^{3/4} < b \sim c \leqslant \varepsilon^{3/5}$ the distribution of pressure disturbance is determined by solving (3.20) and (3.22). Then it is necessary to solve equations for the Prandtl spatial boundary layer for an incompressible gas (3.7)-(3.9) and (3.18) with a prescribed pressure distribution.

Complete systematic analysis of regimes for flow over narrow irregularities with $c \sim O(b)$ indicates that for these irregularities the previous classification scheme for two-dimensional flow regimes given in Fig. 3 is valid.

4. Now let c < b, i.e., consideration be given to flow over irregularities which are narrow and extended in the direction of flow $\varepsilon^2 < a \leqslant c < b \leqslant 1$. It is evident that in this case spreading of gas to the sides will have a considerable effect on the amount of disturbance of flow functions around irregularities.

If irregularity thickness a is so small that disturbance of flow functions is only created as a result of reaction of the irregularity with the boundary-wall part of the boundary layer on the plate, then in a layer of nonlinear disturbances

$$u \sim \Delta u \sim O(a/\varepsilon). \tag{4.1}$$

Since flow should have an essentially spatial character, then from equation for continuity and conservation of a transverse pulse, we obtain

$$w \sim \Delta w \sim O(ac/\varepsilon b),\ \Delta p \sim w^2 \sim O(ac/\varepsilon b)^2. \tag{4.2}$$

In order to study the reaction of an irregularity with a uniform running flow, it is necessary first to consider a disturbed region with typical dimensions $\varepsilon < x \sim y \sim z \sim b \leqslant 1$. However, in the scale of this region an irregularity is a line without thickness or width. Therefore, this region remains undisturbed, and it is necessary to consider region 1 of disturbed flow with typical dimensions $\varepsilon < x \sim b \leqslant 1$, $\varepsilon < y \sim z \sim c < b \leqslant 1$. During reaction of a uniform running flow with an irregularity in this region a vertical velocity is introduced

$$v \sim O(a/b). \tag{4.3}$$

From the continuity equation we have an estimate for disturbance of velocities

$$\Delta u \sim O(a/c),\ w \sim O(a/b), \tag{4.4}$$

and from the equation for conservation of the transverse pulse (considering that disturbance of flow in region 1 should be described by linearized equations relating to a uniform running flow, and therefore $\partial w/\partial x \sim \partial p/\partial z$), we have an estimate for pressure disturbance

$$\Delta p \sim O(ac/b^2). \tag{4.5}$$

In the intermediate case when disturbance of flow functions is created as a result of reaction of an irregularity with the whole boundary layer on a plate and estimates (4.2) and (4.5) are valid simultaneously,

$$ac \sim O(\varepsilon^2). \tag{4.6}$$

Now it is easy to obtain an estimate for the thickness of the layer of viscous nonlinear disturbances δ_1 and pressure disturbance Δ_p for all of the regimes of flow over narrow irregularities:

$$\delta_1 \sim O(\varepsilon c^{1/2}/\Delta p^{1/4}) \sim O(\varepsilon b^{1/3}),\ \Delta p \sim O(c^2/b^{4/3}). \tag{4.7}$$

Estimates (4.1)-(4.7) make it possible to construct a solution of Navier–Stokes equations for the case of flow over narrow irregularities extended in the flow direction.

In layer 3 viscous nonlinear disturbances introduce the following independent variables and asymptotic expansions for flow functions:

$$\begin{gathered} x = bx_3,\ y = \varepsilon b^{1/3} y_3,\ z = cz_3, \\ u = b^{1/3}u_3 + \ldots,\ v = (\varepsilon/b^{1/3})v_3 + \ldots,\ w = (c/b^{2/3})w_3 + \ldots, \\ p = 1/\gamma M_\infty^2 + (c^2/b^{4/3})\,p_3 + \ldots,\ \rho = \rho_w + \ldots,\ \mu = \mu_w + \ldots \end{gathered} \tag{4.8}$$

Substitution of expansions (4.8) in Navier–Stokes equations and completion of the limiting transition with $\varepsilon \to 0$ indicates that region 3 with $a \sim c \sim O(\varepsilon b^{1/3})$, $\varepsilon^{3/2} < b \leqslant 1$ is described to a first approximation by Navier–Stokes equations for an incompressible gas which are parabolic in direction x

$$\frac{\partial u_3}{\partial x_3}+\frac{\partial v_3}{\partial y_3}+\frac{\partial w_3}{\partial z_3}=0,\ \rho_w\left(u_3\frac{\partial u_3}{\partial x_3}+v_3\frac{\partial u_3}{\partial y_3}+w_3\frac{\partial u_3}{\partial z_3}\right)=\mu_w\left(\frac{\partial^2 u_3}{\partial y_3^2}+\frac{\partial^2 u_3}{\partial z_3^2}\right),$$
$$\rho_w\left(u_3\frac{\partial v_3}{\partial x_3}+v_3\frac{\partial v_3}{\partial y_3}+w_3\frac{\partial v_3}{\partial z_3}\right)+\frac{\partial p_3}{\partial y_3}=\mu_w\left(\frac{\partial^2 v_3}{\partial y_3^2}+\frac{\partial^2 v_3}{\partial z_3^2}\right), \tag{4.9}$$
$$\rho_w\left(u_3\frac{\partial w_3}{\partial x_3}+v_3\frac{\partial w_3}{\partial y_3}+w_3\frac{\partial w_3}{\partial z_3}\right)+\frac{\partial p_3}{\partial z_3}=\mu_w\left(\frac{\partial^2 w_3}{\partial y_3^2}+\frac{\partial^2 w_3}{\partial z_3^2}\right),$$

and with $\varepsilon^2 < a \sim c < \varepsilon b^{1/3}$, $\varepsilon^2 < b \leqslant 1$ by Stokes equations [7] (in this case convective terms are dropped in Eq. (4.9). It is evident that in solving the boundary problem (3.5), (3.6), and (4.9) transfer of disturbances upwards through the flow is absent.

If $\varepsilon b^{1/3} < c < b$, then flow in layer 3 to a first approximation will be described by equations for a Prandtl spatial boundary layer for an incompressible gas without the term $\partial p/\partial x$ in the equation for conservation of a longitudinal pulse

$$\frac{\partial u_3}{\partial x_3}+\frac{\partial v_3}{\partial y_3}+\frac{\partial w_3}{\partial z_3}=0,\ \rho_w\left(u_3\frac{\partial u_3}{\partial x_3}+v_3\frac{\partial u_3}{\partial y_3}+w_3\frac{\partial u_3}{\partial z_3}\right)=\mu_w\frac{\partial^2 u_3}{\partial y_3^2},$$
$$\frac{\partial p_3}{\partial y_3}=0,\ \rho_w\left(u_3\frac{\partial w_3}{\partial x_3}+v_3\frac{\partial w_3}{\partial y_3}+w_3\frac{\partial w_3}{\partial z_3}\right)+\frac{\partial p_3}{\partial z_3}=\mu_w\frac{\partial^2 w_3}{\partial y_3^2}. \tag{4.10}$$

Solution of Eqs. (4.10) should satisfy initial boundary conditions (3.9) and the conditions at the surface of an irregularity (3.5) with $a \sim O(\varepsilon b^{1/3})$ or (3.8) [with $\varepsilon b^{1/3} < a \sim O(b^{2/3}c)$].

In order to find external boundary conditions it is necessary now to consider region 2 with characteristic thickness $y \sim O(c)$ with $\varepsilon b^{1/3} < c < \varepsilon$, $\varepsilon^{3/2} < b \leqslant 1$ or $y \sim O(\varepsilon)$ or $\varepsilon \leqslant c < b \leqslant 1$. 1. Therefore, in the first case in region 2 the following independent variables and asymptotic expansions of the flow functions are introduced

$$x_2 = x_3 = x/b,\ y_2 = y/c,\ z_2 = z_3 = z/c,$$
$$u = (c/\varepsilon)Ay_2 + (\varepsilon b^{2/3}/c)u_{22} + \ldots,\ v = (\varepsilon/b^{1/3})v_{22} + \ldots, \tag{4.11}$$
$$w = (\varepsilon/b^{1/3})\,w_{22} + \ldots,\ p = 1/\gamma M_\infty^2 + (c^2/b^{4/3})\,p_2 + \ldots,\ \rho = \rho_w + \ldots,$$

and in the second

$$x_2 = x_3 = x/b,\ y_2 = y/\varepsilon,\ z_2 = z_3 = z/c,$$
$$u = u_0(y_2) + b^{1/3}u_{21} + b^{2/3}u_{22} + \ldots,\ v = (c/b^{1/3})v_{21} + (\varepsilon/b^{1/3})v_{22} + \ldots,$$
$$w = (c/b^{1/3})\,w_{22} + \ldots,\ p = 1/\gamma M_\infty^2 + (c^2/b^{4/3})\,p_2 + \ldots, \tag{4.12}$$
$$\rho = \rho_0(y_2) + b^{1/3}\rho_{21} + b^{2/3}\rho_{22} + \ldots,\ h = h_0(y_2) + b^{1/3}h_{21} + \ldots$$

Substitution of expansions (4.11) and (4.12) in Navier–Stokes equations and completion of the limiting transition with $\varepsilon \to 0$ and $\varepsilon^{3/2} < b \leqslant 1$ indicates that in both cases flow in region 2 to a first approximation will be described by Euler equations without the $\partial p/\partial x$ term in the equation for conservation of a longitudinal pulse linearized relative to the running flow [$u = (c/\varepsilon)Ay_2$ or $u = u_0(y_2)$]. In addition, with $\varepsilon b^{1/3} < c < \varepsilon/b^{1/3}$, $\varepsilon^{3/2} < b \leqslant 1$ use of expansions (4.11) or (4.12) leads to the same basic result: $v_{22} \to 0$ $(y_2 \to 0)$, then external boundary conditions take the form

$$u \to Ay_3,\ v_3,\ w_3 \to 0\ (y_3 \to \infty). \tag{4.13}$$

The boundary problem (3.5), (3.9), (4.10), and (4.13) describes a spatial compensation regime for flow over narrow irregularities with characteristic dimensions $a \sim O(\varepsilon b^{1/3})$, $\varepsilon^{3/2} < b \leqslant 1$, $a < c < \varepsilon/b^{1/3}$. Here due to absence of the $\partial p/\partial x$ term there is no transfer of disturbances upwards through the flow, as with a compensation regime for flow over flat irregularities [21]. With $c \sim O(\varepsilon/b^{1/3})$ for region 2 relationships (3.14) are again valid, and then (3.15) will be the external boundary conditions.

For "thick" narrow irregularities with $a \sim O(b^{2/3}c)$, $\varepsilon^{3/4} < b \leqslant 1$, $\varepsilon/b^{1/3} < c < \varepsilon/b^{2/3}$ in region 2 the following independent variables and asymptotic expansions of flow functions are introduced:

$$x_2 = x_3 = x/b,\ \ y = \varepsilon y_2 + b^{2/3}cf(x_2,\ z_2) + \ldots,\ \ z_2 = z_3 = z/c,$$
$$u = u_0(y_2) + b^{2/3}u_2 + \ldots,\ v = (c/b^{1/3})v_2 + \ldots,\ w = (c/b^{1/3})w_2 + \ldots,$$

$$\rho = \rho_0(y_2) + b^{2/3}\rho_2 + \ldots,\ h = h_0(y_2) + b^{2/3}h_2 + \ldots,$$
$$p = 1/\gamma M_\infty^2 + (c^2/b^{4/3})\,p_2 + \ldots \quad (4.14)$$

Substitution of expansions (4.14) in Navier–Stokes equations and completion of the limiting transition with $\varepsilon \to 0$ and $\varepsilon^{3/4} < b \leqslant 1$ indicates that to a first approximation in region 2 relationships (3.17) are valid, and the solution in region 3 should satisfy external boundary conditions (3.18).

Furthermore it is necessary to consider disturbed region 1 for a uniform running flow with characteristic dimensions $x \sim O(b)$, $y \sim z \sim O(c)$, $\varepsilon^{3/4} \leqslant b \leqslant 1$, $\varepsilon/b^{1/3} \leqslant c \leqslant \varepsilon/b^{2/3}$, in which the following independent variables and asymptotic expansions of flow functions are valid

$$x_1 = x_2 = x_3 = x/b,\ y_1 = y/c,\ z_1 = z_2 = z_3 = z/c,$$
$$u = 1 + b^{2/3}u_1 + \ldots,\ v = (c/b^{1/3})v_1 + \ldots,\ w = (c/b^{1/3})w_1 + \ldots,$$
$$\rho = 1 + (c^2/b^{4/3})\,\rho_1 + \ldots,\ h = 1/(\gamma-1)\,M_\infty^2 + (c^2/b^{4/3})\,h_1 + \ldots,$$
$$p = 1/\gamma M_\infty^2 + (c^2/b^{4/3})\,p_1 \ldots \quad (4.15)$$

Substituting of expansions (4.15) in Navier–Stokes equations and completion of the limiting transition with $\varepsilon \to 0$ and $\varepsilon^{3/4} < b \leqslant 1$ indicates that to a first approximation pressure disturbance in region 1 is described by solution of the boundary problem

$$\partial^2 p_1/\partial y_1^2 + \partial^2 p_1/\partial z_1^2 = 0,\ p_1 \to 0\ (x_1^2 + y_1^2 + z_1^2 \to \infty),$$
$$p_1(x_1, 0, z_1) = p_2(x_2, z_2) = p_3(x_3, z_3), \quad (4.16)$$

which should satisfy internal boundary condition (3.21) with ($c \sim O(\varepsilon/b^{1/3})$) or (3.22) with $\varepsilon/b^{1/3} < c \leqslant \varepsilon/b^{2/3}$. It is evident that solution of boundary problems (3.21), (4.16), or (3.22), (4.16) does not depend on M_∞, in spite of the fact that it disturbs region 1 of a uniform running flow.

Combined solution of boundary problems (3.5), (3.9), (3.15), (4.10) and (3.21), (4.16) describe flow over a narrow irregularity with characteristic dimensions $a \sim O(\varepsilon b^{1/3})$, $\varepsilon^{3/4} < b \leqslant 1$, $c \sim O(\varepsilon/b^{1/3})$ in a free reaction regime. Here due to the absence of the term $\partial p/\partial x$ there is no transfer of disturbances upwards through the flow.

In the case of flow over "thick" narrow irregularities $a \sim O(b^{2/3}c)$, $\varepsilon^{3/4} < b \leqslant 1$, $\varepsilon b^{1/3} < c \leqslant \varepsilon/b^{2/3}$ the distribution of pressure disturbances is determined by solving (3.22), (4.16). Then in region 3 it is necessary to solve the boundary problem (3.8), (3.9), (3.18), (4.10) with prescribed pressure distribution.

5. Shown in Fig. 4 in plan are surfaces in which spatial irregularities with characteristic thickness a, extent b, and width c cause viscous nonlinear disturbances: APNMO is $a \sim O(\varepsilon b^{1/3})$, PQIM is $a \sim O(b^{5/3})$, and AQP is $a \sim O(b^{2/3}c)$. Thicker irregularities cause viscous nonlinear disturbances, and less thick irregularities cause viscous linear disturbances.

The least degenerate regimes for flow over spatial irregularities are realized with $b \sim O(c)$. On line HO ($\varepsilon^2 < a \sim b \sim c < \varepsilon^{3/2}$) flow over irregularities is described by Stokes equations, and at point O ($a \sim b \sim c \sim O(\varepsilon^{3/2})$) it is described by complete spatial Navier–Stokes equations for an incompressible gas. On line OP a spatial compensation regime is realized for flow over irregularities when there is transfer of disturbances upwards through the flow. With $a \sim O(\varepsilon^{5/4})$ and $b \sim c \sim O(\varepsilon^{3/4})$ (point P) a spatial regime is realized for free reaction of flow over irregularities. On line PQ ($a \sim O(b^{5/3}) \sim O(c^{5/3})$) flow over "thick" irregularities is described by Prandtl equations for a spatial boundary layer with prescribed pressure distribution.

For wide irregularities (c > b) in boundary problems there is degeneration along transverse coordinate z, i.e., they break down into a system describing disturbance of flow in plane x, y and containing coordinate z as a parameter, and into linearized equations for conservation of a transverse pulse.

In the case of narrow irregularities (c < b) spreading of gas to the sides becomes a governing factor and there is degeneration of boundary problem along longitudinal coordinate x, i.e., everywhere terms with $\partial p/\partial x$ or $\partial^2 p/\partial x^2$ are dropped. In view of this for all regimes of flow over narrow irregularities there is absence of transfer of disturbances upwards through the flow. In addition, here there is absence of the dependence of solutions on M_∞ even in cases when it disturbs the regions of uniform running flow.

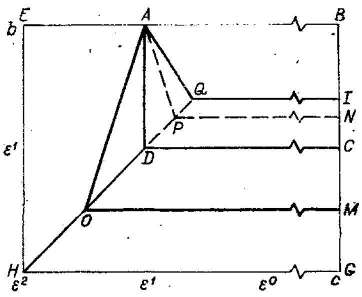

Fig. 4

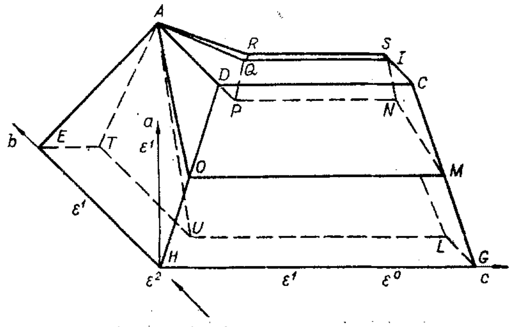

Fig. 5

Surfaces PD, i.e., $a \sim O(\varepsilon^2/c)$ and PNCD, i.e., $a \sim O(\varepsilon^2/b)$, separate irregularities for which disturbances of flow functions are created due to reaction of an irregularity with the boundary-wall part of a boundary layer on a plate, or with a uniform running flow.

It may also be shown that surfaces $a \sim O(\varepsilon b^{2/3})$, $a \sim O(c^2/\varepsilon)$, $a \sim O(b^2)$ and $a \sim O(bc)$ cut off regimes for flow over small surfaces for which pressure gradients $\partial p/\partial x < 1$ or $\partial p/\partial z < 1$ are small, and also convective or diffusion terms in Navier–Stokes equations are small (e.g., $\rho u \partial u/\partial x < 1$ or $\varepsilon^2 \mu \partial^2 u/\partial y^2 < 1$) (these surfaces are not given in Fig. 4).

A general classification scheme for regimes of spatial local flows is given in Fig. 5.

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CALCULATION OF ELECTRON DENSITY IN THE VICINITY OF A BLUNT BODY WITHIN THE FRAMEWORK OF VARIOUS MODELS OF DIFFUSION IN HYPERSONIC FLOW OVER IT

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The question of the influence of the choice of the diffusion model on the distribution of flow parameters in the problem of the flow of a hypersonic air stream over a blunt body is discussed on the basis of a numerical solution of the Navier–Stokes equation.

The high-temperature air in the region of the shock layer between the surface of a body and the bow shock wave (SW) consists of a complicated, multicomponent, partially ionized gas mixture. The solution of the problem of the flow of such mixture over a body within the framework of the complete system of Navier–Stokes equations lies at the limit of the possibilities of modern computers. The question of the degree of complexity of the model which must be used to describe the multicomponent medium is very important from this point of view. In problems of hypersonic air flow over bodies one can construct a hierarchy of models, starting with the most complicated, in which one takes into account 11 components of the mixture reacting with each other (N_2, O_2, NO, N, O, NO^+, N_2^+, O_2^+, N^+, O^+, e), the nonequilibrium of the internal degrees of freedom, and processes of multicomponent diffusion, viscosity, and heat conduction for a sufficiently large number of approximations in Sonine polynomials for the coefficients of transfer of the charged components. The next simpler model has seven components (N_2, O_2, NO, O, N, NO^+, e), in it the internal degrees of freedom are in equilibrium, and transfer processes are taken into account within the framework of the complete system of Navier–Stokes equations. There can be further simplifications of the model, connected with discarding individual terms in the Navier–Stokes equations, as a result of which the type of system changes, with dividing the entire region of flow into subregions, in each of which simpler equations are used (Euler equations, boundary-layer equations), etc. And simplifications are also possible within the framework of any model.

Besides the comparison of the results of the solution with experiment, the comparison with data obtained on the basis of a more complicated model can serve as a criterion for the correctness of the adopted assumptions. A solution has now been obtained within the framework of the seven-component model for the problem of air flow over a blunt body. In the velocity range of 4-6 km/sec at the pressure and densities corresponding to altitudes of 70-100 km above sea level the seven-component model, in which the leading ionization process is associative ionization $N + O \rightleftarrows e + NO^+$, describes the properties of the medium sufficiently well. This model is still complicated for making mass calculations of flow over bodies, however, since each variant of the calculations consumes large amounts of computer time. Below we analyze the possibility of simplifications of the seven-component model of air through approximate allowance for the diffusional properties of the mixture. We consider the question of how the accuracy in assigning the cross sections of elastic collisions of particles of the gas mixture influences the distribution of concentrations of the charged components, and we also investigate the difference arising in the case when multicomponent diffusion is replaced by binary diffusion. In addition, the correctness of the standard assumption that the medium is quasi-neutral is analyzed on the basis of a calculation of the induced electric fields and the space charge in the vicinity of the body. Concrete results

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